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Pricing Arithmetic Asian Put Option with Early Exercise Boundary under Jump-Diffusion Process

Laham, M. F.[1], Ibrahim, S.N.I. *[1,2], and Kilicman, A.[1,2]

[1]*Institute for Mathematical Research, Universiti Putra Malaysia*
[2]*Department of Mathematics, Faculty of Science, Universiti Putra Malaysia*

E-mail: iqmal@upm.edu.my
* *Corresponding author*

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ABSTRACT

Arithmetic Asian options is a financial derivatives whose payoff depends on the average of underlying asset which can either be European-style or American-style. The aim of this study is to provide a pricing formulae for arithmetic Asian option with early exercise boundary under jump-diffusion process by implementing probabilistic approach and conditional expected values. We provide numerical examples for approximation formulae of arithmetic Asian option using quadrature methods and compare the results with Monte Carlo simulation which demonstrate the efficiency of the numerical integration technique.

Keywords: American-Asian option, Option pricing, Monte Carlo Simulation, Jump-diffusion process.

1. Introduction

The Asian option is a path-dependent option whose payoff depends on the average of underlying asset prices over the option's lifetime. Asian options are usually grouped into two main classes according to their payoff. The dependence of the payoff can be specified to a floating strike Asian option when strike price depends on the average price of underlying asset and the fixed strike Asian option as underlying asset is an average. The types of averaging can be classified into geometric and arithmetic basis. Among path-dependent options, the Asian options play an important tool for hedging as this derivative is commonly traded over-the-counter (OTC) in currency and commodity markets; for instance the oil industry.

Essentially, the valuation of path-dependent options involves a non-Markovian systems regarding to the process of asset pricing. Kemna and Vorst (1990) derived closed-form expressions using the partial differential equation (PDE) for geometric average of Asian options. The PDE formulation involves an additional path-dependent state variable. Levy (1992) introduced a closed-form analytic approximation formula for pricing European-style Asian options using Edgeworth series expansion and Wilkinson approximation. For the sake of completeness, we would like to mention more recent work by Han and Liu (2018) give an estimation method to solve a fully nonlinear Partial Differential Equation for Asian options.

It is well known that American-type pricing option with early exercise feature lead to the optimal stopping boundary problems. American-style option gives the holder a right to exercise the option at any time before maturity. The optimal exercise boundary between continuation and stopping regions plays an important role to evaluate American-style options. The option should be exercised when the spot value of the underlying asset is in the stopping region or continue to hold the option otherwise.

Although study on Asian options has been limited to European-style, there are some significant studies on American-style Asian options under the framework of Black-Scholes model by Hansen and Jørgensen (2000), Wu and Fu (2003), and Bokes and Sevcovic (2011). These options differ from ordinary American options in their payoff function which the payoff depends on the price of asset path rather than only on the asset price at the exercise time. In Fusai et al. (2014), they have discussed standard models for pricing Asian options and more recent modelling achievements. Fadugba (2014) pointed out that binomial method for complex option pricing is suffer from computational effort.

Furthermore, jumps-diffusion models are becoming increasingly important in modern options pricing. Merton (1976) introduced a jump process in option pricing to capture discontinuities sample path of underlying assets and leptokurtotic features. Within the jump-diffusion environment, the pricing formulation model becomes a parabolic integro-differential free boundary value problem. Gukhal (2004) derived analytical formula for American options under jump-diffusion environment, where the early exercise premium also contains an additional term due to jumps that is related to the changing cost from the option holder when underlying asset jumps from the stopping area back into the continuation area. While, Shokrollahi and Kılıçman (2015) used a jump mixed fractional Brownian model to pricing option.

In this work, we employ numerical integration techniques to price the arithmetic Asian options. The first approach is quadrature method which was introduced by Sullivan (2000). The second approach utilizes a modified least square Monte Carlo simulation which is used as a benchmark to compare the efficiency of these integration techniques. The numerical quadrature methods are applicable to evaluate option pricing with early exercise features whenever the conditional probability density function is known as demonstrated in Andricopoulos et al. (2007).

Monte Carlo simulation is an important mathematical tool to price options due to its flexibility and well suited method for complex option pricing problem. However, it is challenging to apply simulation on American option, as simulation goes forward in time and establishes an optimal exercise policy. Longstaff and Schwartz (2001) introduced least-squares Monte Carlo method to evaluate American options. de Lima and Samanez (2016) priced an American-Asian option numerically with optimization algorithm using least squares method based on Cerrato et al. (2007) techniques. Knowing the fact that American-style options offer more flexibility and their price should be higher than the price of their equivalent counterpart European option. Monte Carlo simulation is essentially creating this expected value through a stochastic sampling technique.

This paper is organised as follows. In section 2, we present the pricing model for arithmetic Asian option with early exercise boundary under jump-diffusion process. In section 3, we present the numerical pricing results and in section 4, we conclude the paper.

2. Pricing Arithemetic Asian option with Early Exercise Boundary under Jump-Diffusion process

The value of early exercise boundary or American-type option can be estimated by defining an optimal exercising rule on the option and computing the expected discounted payoff function of the option under this policy. The pricing formulation under the jump-diffusion model is a parabolic integro-differential free boundary value problem. The early exercise premium contains an additional term due to occuring jumps, which is related to the adjustment cost made by the option holder when the underlying asset price jumps from the stopping region back into the continuation region.

Now consider the dynamics of asset price process evolves by a continuous component of geometric Brownian motion and a discontinuos component of jumps with a Poisson arrival process. It is the stochastic differential equation (SDE) under risk neutral probability as follows (Merton, 1974);

$$\frac{dS_t}{S_t} = (r - q - \lambda\kappa)dt + \sigma dB_t + (Y-1)dN_t \tag{1}$$

where the American-style option price is written on the underlying asset S_t at time to expiry $\tau = T - t$, where T is the maturity time and t is the current time. Assume the drift coefficient r and the diffusive volatility σ are constants, and q is the dividend yield, where B_t is standard Brownian motion, N_t is a Poisson jump counting process with intensity λ, the relative price jump size $Y - 1$ is a sequence of random variables that are independent identically distributed with the mean $E[Y-1] \equiv \kappa$. Hence, by applying It$\hat{o}$'s formula for jump-diffussion, the solution for SDE is given by;

$$S_t = S_0 \exp\Big[(r - q - \frac{\sigma^2}{2})t + \sigma B_t + \sum_{i=1}^{N_t} Y_i\Big] \tag{2}$$

In order to determine the arithmetic Asian option price with early exercise boundary, the integral equation will be derived. The pricing formulae of continuous-time American-style Asian option problem by the conditional expectation approach is given as follows

$$V(t) = ess \sup_{\tau \in \mathcal{T}(t,T)} \mathbb{E}^P \Big[e^{-r(\tau - t)}(\rho(S_t - A_t)^+ | S_t = S, A_t = A\Big],$$

where $V(t)$ denotes the option value at time t,the elements of $\mathcal{T}$ are stopping times taking values [0,T] and the supremum is achieved by an optimal stopping time T^*. While A_t is a continuous average value of the underlying asset during interval [0,t], while $\rho = 1$ and $\rho = -1$ indicated as call or put option respectively.

In order to reduce the dimension of the pricing formula, Vecer and Xu (2004) changes the probability measure under new equivalent martingale measure $\mathbb{Q}^*$ by

$$\frac{d\mathbb{Q}^*}{d\mathbb{Q}} = e^{-(r-q)T}\frac{S_T}{S_0} = \exp\Big[-\frac{1}{2}\sigma^2 + \sigma B_t + \sum_{i=1}^{N_t} Y_i\Big],$$

and by Girsanov's theorem gives the process $G_t^{Q^*} = G_t^Q - \sigma t$, which is a standard Brownian motion under Q^* measure. The continuous arithmetic average, $A(t)$ can be defined as

$$A(t) = \frac{1}{t}\int_0^t S_u du, \tag{3}$$

and

$$\frac{dA(t)}{A(t)} = \frac{1}{t}(\frac{1}{x(t)} - 1)dt. \tag{4}$$

Now, by defining a new state variable $x(t) = \frac{A(t)}{S(t)}$, they obtained a pricing formula for arithmetic strike American-style Asian options as follows:

$$\begin{aligned}
V_t &= ess \sup_{\tau\in\mathcal{T}_{t,T}} \mathbb{E}^Q\Big[e^{-r(\tau-t)}(\rho(S_\tau - A_\tau)^+\Big] \\
&= ess \sup_{\tau\in\mathcal{T}_{t,T}} \mathbb{E}^{Q^*}\Big[e^{(r-q-r)(\tau-t)}\frac{S_t}{S_\tau}(\rho(S_\tau - A_\tau))^+\Big] \\
&= ess \sup_{\tau\in\mathcal{T}_{t,T}} \mathbb{E}^{Q^*}\Big[e^{-q(\tau-t)}S_t(\rho(1 - \frac{A_\tau}{S_\tau}))^+\Big] \\
&= ess \sup_{\tau\in\mathcal{T}_{t,T}} e^{-q(\tau-t)}S_t\mathbb{E}^{Q^*}\Big[(\rho(1-x))^+\Big]
\end{aligned}$$

Hence, by exercising optimally, the price can be written using the following dynamic programming principle:

$$\begin{aligned}
\tilde{V}_t &= e^{-qt}\frac{V_t}{S_t} \\
&= e^{-q\mathcal{T}^*}\mathbb{E}^{Q^*}\Big[(\rho(1 - x_{\mathcal{T}^*}))^+\Big],
\end{aligned}$$

where $\mathcal{T}^* = \inf(\tau \in [t,T] | x_\tau = x_\tau^*)$ is the optimal stopping time. The function x_t^* is the early exercise boundary, continuously for $t \in [0,T]$ and seperates the state space into two regions.

Definition 2.1 (Bokes and Sevcovic (2011)). *The continuation region, $\mathcal{C}$ and the stopping region $\mathcal{S}$ for American-style option are defined by*

$$\mathcal{C}_{call} = x_t > x_t^*$$
$$\mathcal{S}_{call} = x_t \leq x_t^*$$

The American-style option is optimally exercised when the state of asset prices process $x(\cdot)$ gives a payoff larger than the value function at the first time. The continuation values of an American option is a value of holding rather than stopping or exercising the option. This optimal policy produces early exercise premium feature plus an equivalent European-style option at maturity.

American-style option has a decomposition of an European option value, $\bar{\nu}$ plus the early exercise premium feature, $\bar{e}$ on the underlying asset, x_t paying continuous dividends at a constant rate, q given by:

$$\bar{V}(x(t), t) = \bar{\nu}(x(t), t) + \bar{e}(x(t), t), \tag{5}$$

where:

$$\bar{\nu}(x(t), t) = \mathbb{E}_t^{Q*}[e^{-qT}\rho(1 - x(T))^+], \tag{6}$$

$$\bar{e} = \mathbb{E}^{Q^*}\Big[\int_t^T \rho e^{-qu} x(u)\mu(x(u), u)1_{\mathcal{S}}(x(u))du, \tag{7}$$

$$\mu(x(u), u) = \frac{dA(u)}{A(u)} - (r - q) + \lambda\kappa, \tag{8}$$

$1_{\mathcal{S}}(\cdot)$ is an indicator function for set $\mathcal{S}$, where,

$$1_{\mathcal{S}}(t, y) = \begin{cases} 1 & \text{for } (t, y) \in \mathcal{S}, \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\rho = \begin{cases} 1, & \text{for a call,} \\ -1, & \text{for a put.} \end{cases}$$

Levy (1992) introduced the Wilkinson approximation to approximate the probabilistic distribution of continuous arithmetic averaging. The Wilkinson approximation is given by:

$$\alpha(t, u) = 2\ln \mathbb{E}_t^{Q^*}[x_a(u)] - \frac{1}{2}\ln \mathbb{E}_t^{Q^*}[(x_a(u))^2], \tag{9}$$

and

$$\beta(t, u) = \sqrt{\ln \mathbb{E}_t^{Q^*}[(x_a(u))^2] - 2\ln \mathbb{E}_t^{Q^*}[x_a(u)]}. \tag{10}$$

Using Equations (7) and (8), the first and second conditioned moments of $x_a(t)$ can be derived analytically as shown in Lemma 1 by Hansen and Jørgensen (2000).

Lemma 2.1. *For $u > t$, we consider $\ln x_a(t,u)|\mathcal{F}_t$ is a log-normal distribution with mean, $\alpha_a(t,u)$ and its variance, $\beta_a^2(t,u)$, the first and second conditioned moments of $x_a(u)$ under probability Q^* can be calculated as follows*

$$\begin{aligned}\mathbb{E}_t^Q[x_a(u)] &= x_a(t)\frac{t}{u}e^{-(r-q)(u-t)} \\ &\quad + \frac{1}{(r-q)u}(1-e^{-(r-q)(u-t)}),\end{aligned}$$

$$\begin{aligned}\mathbb{E}_t^Q[x_a^2(u)] &= x_a^2(u)(\frac{t}{u})^2 e^{-(2r-2q-\sigma^2)(u-t)} \\ &\quad + x_a(t)\frac{2te^{-(r-q)(u-t)}}{u^2(r-q)}(1-e^{-(r-q)(u-t)}) \\ &\quad + \frac{(r-q-\sigma^2)-2(r-q-\frac{\sigma^2}{2})}{u^2(r-q)(r-q-\frac{\sigma^2}{2})(r-q-\sigma^2)} \\ &\quad + \frac{(r-q)e^{-(2r-2q-\sigma^2)(u-t)}}{u^2(r-q)(r-q-\frac{\sigma^2}{2})(r-q-\sigma^2)}.\end{aligned}$$

Lemma 1 leads to the pricing formulae for arithmetic Asian options with early exercise boundary under jumps-diffusion process as provided in the following proposition.

Proposition 2.1. *The pricing formulae for an arithmetic Asian options with early exercise boundary under jump-diffusion boundary, $\bar{V}(x(t),t)$ is given by:*

$$\bar{V}(x(t),t) = \bar{\nu}(x(t),t) + \bar{e}(x(t),t)$$

where:

$$\begin{aligned}\bar{\nu}(x(t),t) &= \sum_{n=0}^{\infty}\rho\frac{e^{-qT}\lambda^n}{n!}\Big[\Phi\big(-\rho\frac{\alpha(t,T)}{\beta(t,T)}\big)\Big] \\ &\quad - e^{-\alpha(t,T)+\frac{\beta(t,T)}{2}}\Phi\Big(-\rho\big(\frac{\alpha(t,T)}{\beta(t,T)}+\beta(t,T)\big)\Big),\end{aligned}$$

$$\bar{e} = \int_t^T \rho \frac{e^{-qu}\lambda^n}{n!} \Big[(q + \frac{1}{u})\Phi(\rho(\beta(t,u) - \gamma(t,u))) \\ - (r + \frac{1}{u})e^{\alpha + \frac{\beta^2}{2}} \Phi(-\rho\gamma(t,u)) \Big] du,$$

and Φ is the standard normal cumulative distribution function.

Proof. Consider lognormal conditioned distribution, $\ln x(u) \sim N(\alpha(t,u), \beta^2(t,u))$ at time t, and we have

$$x(t) = \frac{\frac{1}{t}\int_0^t S(t)dt}{S(t)},$$

then the evolution of $x(\cdot)$ by Ito's Lemma becomes:

$$dx(t) = x(t)\Big[\frac{dA(t)}{A(t)} - r + \lambda\kappa - \sigma dB^{Q'}(t)\Big].$$

Now, by substitute expression in (6) and (7) using (9), (10) and Lemma 1, we can approximate the probabilistic distribution for continuous arithmetic average. Then we have proof the Proposition 1 as follow:

$$\bar{\nu}(x(t),t) = \mathbb{E}_t^{Q*}[e^{-qT}\rho(1 - x(T))^+], \\ = \sum_{n=0}^{\infty} \rho \frac{e^{-qT}\lambda^n}{n!} \Big[\Phi(- \rho\frac{\alpha(t,T)}{\beta(t,T)}) \Big] \\ - e^{-\alpha(t,T) + \frac{\beta(t,T)}{2}} \Phi\Big(- \rho(\frac{\alpha(t,T)}{\beta(t,T)} + \beta(t,T))\Big)$$

for the European part and the American-style early exercise as follow:

$$\bar{e} = \mathbb{E}^{Q^*}\Big[\int_t^T \rho e^{-qu} x(u)\mu(x(u),u) 1_{\mathcal{S}}(x(u))du, \\ = \mathbb{E}^{Q^*}\Big[\int_t^T \rho e^{-qu} x(u) 1_{\mathcal{S}}(x(u)) \\ \Big(\frac{dA(u)}{A(u)} - (r-q) + \lambda\kappa\Big)du\Big], \\ = \int_t^T \rho \frac{e^{-qu}\lambda^n}{n!} \Big[1_{\rho x^* > \rho x}\Big(\frac{1}{u}(1 - x(u)) - rx(u) + q\Big)\Big], \\ = \int_t^T \rho \frac{e^{-qu}\lambda^n}{n!} \Big[(q + \frac{1}{u})\Phi(\rho(\beta(t,u) - \gamma(t,u))), \\ - (r + \frac{1}{u})e^{\alpha + \frac{\beta^2}{2}} \Phi(-\rho\gamma(t,u)) \Big] du.$$

□

Given the pricing formulae of arithmetic Asian options with early exercise boundary under jumps-diffusion process in Proposition 1, we can let $\rho = 1$ to obtain the price of call option or $\rho = -1$ to obtain the price for a put option. In this paasset price jumpsper, we choose $\rho = -1$.

2.1 Quadrature Method

The quadrature method allows numerical integration to directly evaluate conditional expectation representing option prices with early exercise boundary. A discretization scheme is used in order to compute the early exercise boundary, x_t^*at each time interval.

We divided the time interval $[0, T]$ into M subintervals of equal lengths $\Delta t = t_k - t_{k-1}$, for $k = 1, 2, ..., M$. The value of the boundary $x^*(T)$ at the maturity is 1. Since $t_M = T$ then $x^*(t_M) = 1$. Hence, the boundary condition $x^*(t_{M-1})$ can be calculated as follows:

$$\rho(1 - x^*(t_{M-1})) = \bar{\nu}(x^*(t_{M-1}), t_k) + \int_t^T f(x^*(t_{M-1}), x^*(\tau), \tau) d\tau.$$

We employ the trapezoidal rule to compute the integral in the early exercise premium, $f(x(t), x^*(t), t, \tau)$ as follows:

$$\begin{aligned} \rho(1 - x^*(t_{M-1})) = &\ \bar{\nu}(x^*(t_{M-1}), t_M, T) \\ &+ \frac{\Delta T}{2}\Big[f(x^*(t_{M-1}, x^*(t_M), T) \\ &+ f(x^*(t_{M-1}), x^*(t_{M-1}), T)\Big] \end{aligned}$$

which gives the solution of $x^*(t_{M-1})$. Similarly, we can compute the values $x^*(t_{k-1})$ for $k = M-2, M-3, ..., 0$ using trapezoidal rule where the method works backwards toward $t = 0$. The options value can be determined by final numerical evaluation of the integral when $t = 0$ and $x^*(0)$ is determined.

2.2 Modified Least Square Monte Carlo Simulation

Evaluating American-style Asian options through some modification of Monte Carlo simulation method involves dynamic programming and approximation functions. Thus, we must approximate with a finite number of exercise times under Monte Carlo simulation denoted by N. If the path is in the exercise region at any of these times, then the option is exercised. The regression method is used to estimate the optimal coefficients of the approximation when the option is in the money. The Euler discretization scheme has been used for independent normally distributed random variables Z_1, Z_2 and a Poisson random variable N with intensity λ to approximate the asset price process under jump-diffusion model as follows:

$$S(t+\Delta) = S(t) + (r-q-m)\Delta + \sigma\sqrt{\Delta}Z_1 + N\mu_J + \sqrt{N}\sigma_J Z_2, \qquad (11)$$

where $\Delta t = t_i - t_{i-1}$ is the time interval size, which is assumed to be constant and we simulated $X(t_{i-1}) = \log S(t_{i-1})$. Here $N\mu_J + \sqrt{N}\sigma_J Z_2$ reflects the changes due to jumps on $[t, t+\Delta]$.

The least square Monte Carlo method allows approximate the continuation value by using regression to possibly exercise the option. Thus, the continuation values $C_i(s)$ is given by:

$$\begin{aligned} C_i(s) &= e^{-r(t_{i+1}-t_i)}\mathbb{E}[V_{i+1}|S_{i+1}] \\ &= \sum_{j=1]}^{J} \beta_{ij}\chi_j(s) \\ &= \beta_i^\top \chi(s), \end{aligned}$$

where $\chi_j(s)$ is a vector of some basis function and β_i is the vector of regression coefficients using least squares rule which depends on time t_i. The least squares estimation of the regression coefficients, β_i is given by

$$\hat{\beta} = \hat{B}_{\chi,i}^{-1}\hat{B}_{\chi V,i}, \qquad (12)$$

where

$$\hat{\beta}_{\chi,i} = \frac{1}{N}\sum_{n=1}^{N} \chi(S_i^n)\chi(S_i^n)^\top,$$

$$\hat{\beta}_{\chi V,i} = \frac{1}{N}\sum_{n=1}^{N} \chi(S_i V_{i+1}^n)\chi(S_{i+1}^n),$$

The weighted Laguerre polynomial is employed for the least squares based regression function, $\chi_i(x)$ to estimate the optimal coefficients for the approximation which are explicitly defined as:

$$\chi_j(x) = e^{-\frac{x}{2}}L_k(x), k = j-1, \qquad (13)$$

where $L_k(x)$ is the Laguerre polynomials defined as:

$$L_k(x) = \frac{e^x}{k!}\frac{d^k}{dx^k}(e^{-x}x^k)$$
$$= \begin{cases} 1, & k=0 \\ 1-x, & k=1 \\ \frac{1}{k}[(2k-1-x)L_{k-1}(x)]-(k-1)L_{k-2}(x), & k \geq 1 \end{cases}$$

By applying stopping strategy and simulating the Monte Carlo path, then the value of the option can be estimated as

$$\hat{V} = \frac{1}{N}\sum_{n=1}^{N} V_0^n \tag{14}$$

where $V_0(n) = \mathbb{E}[h_i(S_i^n)]$, and h be some payoff function. The procedure for pricing arithmetic Asian options with Early Exercise Boundary under jumps-diffusion process using modified least-squares Monte Carlo simulation is summarize in the algorithm 1.

Algorithm 1 Modified Least-Squares Monte Carlo Simulation.

1. Load regression coefficients $\chi_i, i = 1, ..., M$.
2. Simulate N independent paths of the asset price process under jump-diffusion model $\{S_1(n), S_2(n), ..., S_M(n)\}$, $n = 1, ..., N$
3. Apply forward induction for $i = 1, ..., M-1$ and for each $n = 1, ..., N$.
   - Compute continuation values $C_i(S_i) = \beta_i^\top \chi(S_i)$.
   - Compute the payoff functions $h_i(S_i^n)$.
4. For $i = M$, set $C_M(S_M(n)) = 0$ and compute $h_M(S_M(n))$.
5. Set $V_0^n = h_{i^*}(S_{i^*}(n))$, $i^* = \min[i \in \{1, ..., M\} : h_i(S_i^n \geq C_i(S_i^n)]$.
6. Compute the estimated option value as $V_0 = \frac{1}{N}\sum_{n=1}^{N} V_0^n$.

3. Numerical Results

In this section, we document the computation result of the arithmetic Asian put option with early exercise boundary under jump-diffusion process. We price the put option using numerical integration of quadrature approximation formulae and the modified least squares Monte Carlo simulation. We consider arithmetic Asian put options by letting $\rho = -1$ in the formulae from Proposition 1 and we price the option for a range of underlying assets, $S_0 = 5, 10, 15, 20, 25, 30, 35$ with the inputs in Table 1. [1]

Table 1: Inputs to price the Asian options.

Inputs	Values
Strike price, K	20
Interest rate, r	0.03
Dividend rate, q	0.04
Volatility σ	0.04
Maturity, T	1
Jumps intensity, λ	0.4
Jumps drift, μ_J	-0.95
Jumps volatility, σ_J	0.5

We let the number of basis functions fixed to $J = 5$, the number of simulation, $N = 10000$ and the number of time steps, $M = 2^5 = 32$ within the Monte Carlo simulation method. Table 2 documents the prices of an arithmetic Asian put option with early exercise boundary under jump-diffusion process for a range of asset prices using numerical integration quadrature method denoted by $\bar{V}$ and modified least square Monte Carlo simulation. We also plot the price convergence of the Monte Carlo simulation in Figure 1. On the other hand, the prices of arithmetic Asian put option with early exercise boundary without jump-diffusion process is provided in Table 3. It can be clearly seen that the option prices with jumps are cheaper than the option prices without jumps.

Table 2: Prices of arithmetic Asian put option with early exercise boundary under jump-diffusion process.

		Modified LS Monte Carlo Simulation Prices					
S_0	$\bar{V}$	N = 1000	N=5000	N=10000	N = 100000	N = 500000	RMSE
5	15.2012	15.0014	15.1011	15.9540	15.0078	15.2141	0.3617
10	9.9999	9.4404	9.3719	10.7938	9.3715	9.2984	0.6670
15	5.6287	5.4967	5.4824	6.0026	5.5079	5.4725	0.3335
20	3.2876	3.2648	3.2391	3.2594	3.2686	3.2477	0.3680
25	1.6709	2.0400	2.0219	2.0386	2.0527	2.0408	0.7682
30	0.6767	1.4613	1.4307	1.4443	1.4466	1.4415	0.7682
35	0.4072	1.1860	1.1753	1.1712	1.1729	1.1708	0.7576

[1] The computations were implemented in MATLAB and conducted on an Intel Core i5-2410M processor @ 2.30GHz machine running under Windows 7 Home Premium with 4GB RAM

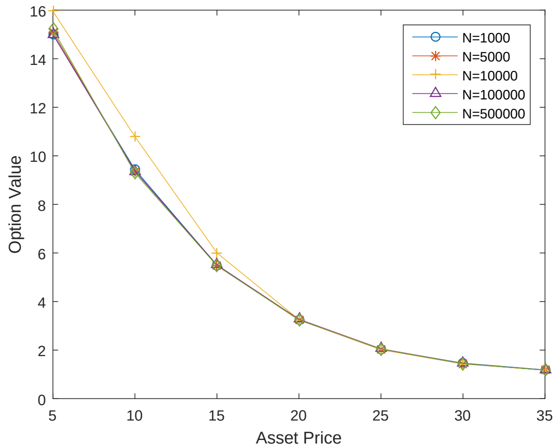

Figure 1: Price convergence of Monte Carlo simulation.

Table 3: Prices of arithmetic Asian put option with early exercise boundary.

		Modified LS Monte Carlo Simulation Prices					
S_0	$\bar{V}$	N= 1000	N=5000	N=10000	N = 100000	N = 500000	RMSE
5	15.9999	16.1335	16.0857	16.0744	16.0846	16.0822	0.0946
10	10.2898	11.2918	11.1975	11.1747	11.1946	11.1900	0.9209
15	6.1129	6.5838	6.4686	6.4399	6.4646	6.4560	0.3733
20	2.8929	2.9823	2.9368	2.9399	2.9485	2.9446	0.0598
25	1.4137	1.4574	1.4252	1.4341	1.4325	1.4329	0.0252
30	0.8279	1.0510	1.0466	1.0482	1.0458	1.0463	0.2197
35	0.3228	0.9824	0.9834	0.9823	0.9816	0.9814	0.5980

*RMSE = Root mean-squared relative option pricing errors

4. Conclusion

In this paper, the fundamental premise of the study was to provide a pricing framework for arithmetic Asian option under jump-diffusion process and to extend the pricing of arithmetic Asian option with early exercise boundary using numerical integration of quadrature methods. A modification of least-squares Monte Carlo simulation to price Asian options with early exercise boundary is also as part of contribution. It shows that the numerical integration of quadrature methods and the modified least-squares Monte Carlo simulation can be applied to price the arithmetic Asian option with early exercise boundary under jump-diffusion process. In the future, we plan to price the arithmetic Asian option with early exercise boundary under jump-diffusion process using finite difference tecniques.

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